WJEC
CBAC

# Level 2 Certificate
## Specification for teaching from 2010

# Additional Mathematics

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**WJEC LEVEL 2 CERTIFICATE**
**in ADDITIONAL MATHEMATICS**

## For Teaching from 2010
## For Award from 2011


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# ADDITIONAL MATHEMATICS

# SUMMARY OF ASSESSMENT

| **Additional Mathematics** | **100 marks** | **duration 2½ hours** |
|---|---|---|
| Calculators are allowed.<br>Candidates are expected to answer all questions. | | |

The specification is designed to stretch the most able candidates for GCSE Mathematics. It also provides an appropriate and fulfilling course for those who have sat GCSE Mathematics a year early or are planning to follow a mathematics course at AS level or a related discipline post-16.

It is also envisaged that the course will strengthen considerably the formal techniques necessary for further study and hence narrow the perceived gap between GCSE and post-16 qualifications.

The scheme is not tiered and a candidate's result will be reported as distinction, merit or pass. Candidates failing to achieve a pass will not receive an award.

The examination will be available in the summer series only. The first examination will be held in June 2011 and subsequent examinations will be held in June of each year.

The assessment will take into account the quality of written communication (including mathematical communication) used in the answers to specific questions. These questions will be clearly indicated on the question paper.

| | | |
|---|---|---|
| Entry code | 9550 01 | (English medium) |
| | 9550 W1 | (Welsh medium) |

**Qualification Accreditation Number: 500/7929/3**

# ADDITIONAL MATHEMATICS

## 1 INTRODUCTION

### 1.1 Rationale

Although all GCSE qualifications in mathematics require students to study algebraic, geometric and trigonometric techniques, it has been apparent that many centres have been searching for ways to reinforce these basic mathematical techniques and develop understanding and proficiency in dealing with formal proofs in mathematics. Fluency in these techniques is an essential requirement for further study in mathematics and related disciplines. This reinforcement ensures that the perceived gap between the skills and techniques addressed in a GCSE course in mathematics and those required for AS mathematics and related disciplines is reduced.

This qualification will also provide an appropriate course of study for the most able candidates to be stretched in their mathematics curriculum and will also provide an appropriate course of study for candidates who acquire GCSE Mathematics early.

### 1.2 Aims and Learning Outcomes

This WJEC Level 2 Certificate in Additional Mathematics is based on needs identified through a pilot Additional Mathematics qualification. It is accredited for use in Wales only.

WJEC Level 2 Certificate in Additional Mathematics should encourage students to be challenged by following a satisfying and worthwhile course of study which takes students further than a GCSE course in mathematics. The course should help learners to develop confidence in, and a positive attitude towards mathematics. This specification should prepare learners to make informed decisions about further learning opportunities and career choices.

A course based on this specification must enable students to:

- develop a positive attitude to mathematics;
- formulate and reinforce manipulative algebraic and trigonometric skills which are contained in the GCSE Mathematics specification;
- develop knowledge, skills and understanding of mathematical methods and concepts which are further to those contained in a GCSE Mathematics specification;
- acquire and use problem-solving strategies;
- be well prepared for the further study of mathematics and other disciplines;
- be creative in mathematics and have the confidence to engage with challenging problems and novel and abstract situations;
- reason mathematically, make deductions and inferences, draw conclusions and engage with formal mathematical proof;
- develop an awareness of the holistic nature of mathematics.

## 1.3 Prior Learning and Progression

Although there is no specific requirement for prior learning, this specification builds upon the knowledge, skills and understanding developed in the Key Stage 3 and Key Stage 4 Programmes of Study in Mathematics as defined by the National Curriculum.

This specification provides a basis for the study of mathematics and related subjects at Advanced Subsidiary and Advanced GCE and also for further study leading to other qualifications. It is specifically designed to narrow the perceived gap between GCSE and AS courses in mathematics.

## 1.4 Equality and Fair Assessment

This specification has been designed to offer fair access for all candidates and to minimise any later need to make reasonable adjustments for candidates who have particular requirements, while preserving the rigour of the qualification.

A review of the qualification and the regulatory criteria on which it is based has not revealed any potential barriers to access arising from the assessment of skills and understanding in the subject.

Reasonable adjustments are made for disabled candidates in order to enable them to access the assessments. For this reason, very few candidates will have a complete barrier to any part of the assessment. Information on reasonable adjustments is found in the Joint Council for Qualifications document *Regulations and Guidance Relating to Candidates who are eligible for Adjustments in Examinations.* This document is available on the JCQ website (www.jcq.org.uk).

Candidates who are still unable to access a significant part of the assessment, even after exploring all possibilities through reasonable adjustments, may still be able to receive an award. They would be given a grade on the parts of the assessment they have taken and there would be an indication on their certificate that not all of the competences have been addressed. This will be kept under review and may be amended in future.

## 1.5 Classification Codes

This specification does not overlap with any other Level 2 qualification offered by WJEC and there are no restrictions on concurrent entry for other qualifications.

Centres should be aware that candidates who enter for more than one qualification with the same classification code will only have one award (the highest) counted for the purpose of the School and College Performance Tables.

The classification code for this specification is 2340.

# 2 SPECIFICATION CONTENT

Candidates will be expected to apply their knowledge, skills and understanding of the subject content listed below in abstract and real-life contexts and to demonstrate strategies for problem-solving.

The subject content for WJEC Level 2 Certificate in Additional Mathematics consists of:

- Algebra
- Coordinate geometry
- Mensuration
- Calculus
- Trigonometry

## ALGEBRA

| **Topic** | **Notes and examples** |
|---|---|
| Simplify numerical expressions involving surds. | Simplification of expressions involving surds.<br>$(\sqrt{3}+\sqrt{2})^2-(\sqrt{3}-\sqrt{2})^2=4\sqrt{6}$<br>Including the rationalisation of the denominator of an expression such as $\frac{1}{(2-\sqrt{3})}$. |
| Understand and use indices with negative and fractional values. | *Find the values of*<br>$8^{\frac{2}{3}}$ $16^{-\frac{1}{4}}$ $7^0\times 16^{\frac{3}{4}}$.<br>*Simplify*<br>$(y^{\frac{2}{3}}\times y^{\frac{1}{2}}\times y^{\frac{5}{6}})^{\frac{1}{2}}$ $\frac{5x^{\frac{1}{2}}+6x^{\frac{3}{2}}}{7x^{\frac{1}{2}}}$ |
| Factorisation of quadratic expressions. | Factorisation of quadratic expressions of the form $ax^2 + bx + c$, $a \neq 1$.<br>*Factorise (i)* $3x^2 - 48$<br>*(ii)* $3m^2 - 10m + 3$ *(iii)* $12d^2 + 5d - 2$. |
| Solution of quadratic equations by factorisation. | The solution by factorisation of quadratic equations of the form $ax^2 + bx + c = 0$, $a \neq 1$. |
| Formation and manipulation of quadratic equations. | *A rectangle has longer side (2x + 3) m and shorter side (x + 1) m. Its area is 66 m$^2$.*<br>*Write down a quadratic equation which is satisfied by x and arrange it in the form $ax^2 + bx + c = 0$ where a, b and c are integers.* |
| Simplify algebraic expressions involving fractions. | Simplify algebraic expressions of the form<br>$\frac{a}{dx \pm e} \pm \frac{b}{fx \pm g} \pm \frac{c}{h}$ |
| Solution of algebraic equations involving fractions. | Solution of equations of the form<br>$\frac{a}{dx \pm e} \pm \frac{b}{fx \pm g} = \frac{c}{h}$ |

| **Topic** | **Notes and examples** |
|---|---|
| Completing the square. | Deriving roots of general quadratic equations.<br>*Show that the roots of the quadratic equation*<br>$ax^2 + bx + c = 0$ *are* $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.<br><br>Solving quadratic equations by completing the square.<br>*Use the method of completing the square to solve* $x^2 + 5x - 3 = 0$. *Express your solutions in surd form.*<br><br>*By completing the square find the minimum value of* $x^2 - 16x + 61$. |
| Remainder theorem.<br><br><br><br><br><br>Factor theorem. | *Find the remainder when* $2x^3 + x - 5$ *is divided by* $2x - 1$.<br><br>*Show that* $x - 1$ *is a factor of* $x^3 - 7x + 6$.<br><br>*When* $x^3 + 2x^2 + a$ *is divided by* $x - 2$ *the remainder is* $-3$. *Find* $a$.<br><br>Factorise polynomials of at most degree three.<br><br>*Show that* $x - 1$ *is a factor of* $2x^3 - x^2 - 2x + 1$, *and hence factorise the expression.* |
| Algebraic proof. | Algebraic identities.<br>Use of the symbol ≡<br><br>*Show that*<br>$(3x - 1)(3x + 1) - (1 - x)(1 + x) + 3(1 - 2x)(1 + 2x) \equiv 1 - 2x^2$<br>*Show that*<br>$(x - 1)(x^2 + 2x + 3) - x(x + 1) \equiv x^3 - 3$<br><br>*Prove that the sum of any three consecutive numbers is divisible by* 3. |
| Solution of one linear and one quadratic equation. | Solution of equations of the form<br><br>$ax^2 + by^2 + cxy + dx + ey + f = 0$ and $px + qy + r = 0$,<br><br>where some of the coefficients may be zero. |

## COORDINATE GEOMETRY

| Topic | Notes and examples |
|---|---|
| Distance between two points.<br>Gradient of the straight line joining two points. | Two dimensions only. |
| Equation of a straight line in the forms<br>$y = mx + c$<br>$ax + by + c = 0$<br>$y - y_1 = m(x - x_1)$ | Use of the fact that $m$ is the gradient and $c$ is the intercept on the $y$-axis. |
| Equations of lines parallel and perpendicular to a given line. | Use of the fact that parallel lines have equal gradients and the product of the gradients of two perpendicular lines is – 1. |
| Intersection of a straight line with a curve. | The coordinates of the points of intersection of a curve of the form $ax^2 + by^2 + cxy + dx + ey + f = 0$ with a line of the form $px+qy+r = 0$. |

## MENSURATION

| Topic | Notes and examples |
|---|---|
| Measurement including distances and angles, to more complex plane shapes and solids, including circular arcs, cylinders, cones and spheres. | Volumes and surface areas of spheres, cones and pyramids.<br>Lengths of circular arcs.<br>Areas of sectors and segments of circles. |

**CALCULUS**

| **Topic** | **Notes and examples** |
|---|---|
| Given $y = f(x)$ find $\frac{dy}{dx}$ from first principles in simple cases. | $f(x)$ will be linear or quadratic. |
| Differentiation of $x^n$ and related sums and differences. | Use of $\frac{d(x^n)}{dx} = nx^{n-1}$<br><br>*Differentiate*<br>$f(x) = 2x^8$, $f(x) = \frac{3}{4x^2}$, $f(x) = x^{\frac{1}{2}}$, $f(x) = 3x^{-\frac{1}{2}}$<br>$f(x) = (x-3)^2$, $f(x) = 4x^3 - 2x + 3$, $f(x) = 2x^{\frac{1}{2}} - x^{-\frac{1}{2}}$ |
| Second derivatives in simple cases. | *Given* $y = x^3 + 3x^2 + 1$ *find* $\frac{d^2y}{dx^2}$. |
| Equations of tangents. | *Find the equation of the tangent to the curve* $y = x^2$ *at the point (2,4).* |
| Maximum and minimum values.<br><br>(Consideration of points of inflexion will not be assessed.) | Understand that $\frac{dy}{dx} = 0$ at maximum and minimum points on a curve.<br><br>*Find the maximum and minimum points on the curve* $y = x^3 + 3x^2 + 1$ *and determine their nature. Hence sketch the curve.* |
| Integration as the reverse of differentiation. | Given that $\frac{dy}{dx} = x^n$ then $y = \frac{x^{n+1}}{n+1} + c$ $(n \neq -1)$ |
| Indefinite integrals as the reverse of differentiation. | $\int \frac{1}{x^2}\,dx$, $\int 8\sqrt{x}\,dx$, $\int (3x^2 + 5x - 2)\,dx$, $\int \left(1 - \frac{2}{x^2}\right) dx$. |
| Evaluation of definite integrals. | $\int_0^3 8\sqrt{x}\,dx$, $\int_1^2 x^3 + 4\,dx$. |
| Applications to simple areas, where the curve is entirely above or below the $x$-axis in the given interval. | The required area will be enclosed between the curve and lines drawn parallel to either the $x$- or $y$- axis.<br>*Find the area under the curve* $y = x^2$ *between* $x = 1$ *and* $x = 2$.<br>*Find the area between the curve* $y = x^2 + 6x$ *and the x-axis for values of x from* $-3$ *to 0.* |

**TRIGONOMETRY**

| **Topic** | **Notes and examples** |
|---|---|
| Trigonometric ratios of angles up to four right-angles. | The application and understanding of the sine, cosine and tangent of angles between 0° and 360°. |
| Special values for 0°, 30°, 45° and their multiples. | Use of the special values of sine, cosine and tangent of 0°, 30° and 45° and their multiples. |
| Simple problems in three-dimensions. | The application of Pythagoras' theorem, the trigonometry of right-angled triangles, the sine rule and the cosine rule to problems in three-dimensions. (The ambiguous case of the sine rule will not be assessed.) |
| The graphs and behaviour of trigonometric functions, and the application of these to the solution of simple equations. | Sketching trigonometric graphs of the form $y = a\sin k\theta$, $y = b\cos k\theta$ and $y = c\tan k\theta$.<br><br>Solutions of equations of the form $a\sin k\theta = b$, $a\cos k\theta = b$, $a\tan k\theta = b$.<br>*Find the solutions of the equation* $2\cos 3\theta = -1$ *in the range* $0° \leq x \leq 180°$ |

# 3 ASSESSMENT

## 3.1 Scheme of Assessment

The scheme of assessment will consist of 1 written paper in which a calculator will be allowed.

The maximum mark on the paper will be 100.

The written paper will be $2\frac{1}{2}$ hours in duration and will be in the form of a question/answer booklet.

Candidates will be expected to attempt all the questions.

The examination will be available in the summer of each year.

The result of each candidate will be reported as distinction, merit or pass.

## 3.2 Resources

**Formulae Lists**

Candidates will be expected to know the formulae given in the Higher Tier GCSE Mathematics examination papers. Candidates will not be given these or any other formulae in the Additional Mathematics examination.

## 3.3 Assessment Objectives

The specification requires candidates to demonstrate their knowledge, skills and understanding in the following assessment objectives. These relate to the knowledge, skills and understanding in the relevant programme of study.

**AO1 Recall and use their knowledge of the prescribed content.**

**AO2 Select and apply mathematical methods.**

**AO3 Interpret and analyse problems and use mathematical reasoning to solve them.**

The written paper will assess all assessment objectives.

The weightings of the assessment objectives will be within the following ranges.

| ASSESSMENT OBJECTIVES | | Weighting (%) |
|---|---|---|
| AO1 | Recall and use their knowledge of the prescribed content | 60 – 70 |
| AO2 | Select and apply mathematical methods | 15 – 25 |
| AO3 | Interpret and analyse problems and use mathematical reasoning to solve them | 10 – 20 |

## 3.4 Quality of Written Communication

The assessment will take into account the quality of written communication (including mathematical communication) used in the answers to specific questions. These questions will be clearly indicated on each question paper.

Mark schemes for all components include the following specific criteria for the assessment of written communication (including mathematical communication):

- legibility of text; accuracy of spelling, punctuation and grammar; clarity of meaning;
- selection of a form and style of writing appropriate to purpose and to complexity of subject matter;
- organisation of information clearly and coherently; use of specialist vocabulary where appropriate.

# 4 AWARDING AND REPORTING

WJEC Level 2 Certificate in Additional Mathematics is not tiered and is reported as distinction, merit or pass, where distinction is the highest award. The attainment of pupils who do not succeed in reaching the lowest possible standard to achieve a pass is recorded as unclassified and they will not receive a certificate.

# 5 GRADE DESCRIPTIONS

Grade descriptions are provided to give a general indication of the standards of achievement likely to have been shown by candidates awarded particular grades. The descriptions must be interpreted in relation to the content specified by the specification; they are not designed to define that content. The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of the examination may be balanced by better performances in others.

**Distinction**

Candidates recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use.

Candidates manipulate algebraic and trigonometric expressions and equations, and use graphs, sketches and diagrams, all with high accuracy and skill. They use mathematical language correctly and proceed logically and rigorously through extended arguments or formal proofs. When confronted with unstructured problems they can often devise and implement an effective and efficient solution strategy. If errors are made in their calculations or logic, these are sometimes noticed and corrected.

Candidates recall or recognise almost all the standard algorithms that are needed, and select appropriate ones to solve problems.

Candidates make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are aware of any limitations to their use. They present results to an appropriate degree of accuracy.

**Merit**

Candidates recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use.

Candidates manipulate algebraic and trigonometric expressions and equations, and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill. They use mathematical language with some skill and sometimes proceed logically through extended arguments or formal proofs. When confronted with unstructured problems they sometimes devise and implement an effective and efficient solution strategy. They occasionally notice and correct errors in their calculations.

Candidates recall or recognise most of the standard algorithms that are needed, and usually select appropriate ones to solve problems.

Candidates usually make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are sometimes aware of any limitations to their use. They usually present results to an appropriate degree of accuracy.

**Pass**

Candidates recall or recognise some of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use.

Candidates manipulate algebraic and trigonometric expressions and equations, and use graphs, sketches and diagrams, all with some accuracy and skill. They sometimes use mathematical language correctly and occasionally proceed logically through extended arguments or formal proofs.

Candidates recall or recognise some of the standard algorithms that are needed, and sometimes select appropriate ones to solve problems.

Candidates often make appropriate and efficient use of contemporary calculator technology and other permitted resources. They often present results to an appropriate degree of accuracy.

# 6 THE WIDER CURRICULUM

## 6.1 Key Skills and Essential Skills (Wales)

WJEC Level 2 Certificate in Additional Mathematics will provide a range of opportunities for developing these skills, by providing contexts in which evidence for key skills or essential skills (Wales) portfolios may be produced. The following key/essential skills can be developed through this specification at level 2:

- Communication
- Application of Number
- Information and Communication Technology
- Problem Solving
- Working with Others
- Improving Own Learning and Performance

Mapping of opportunities for the development of these skills against key/essential skills evidence requirements at level 2 is provided in 'Exemplification of Key/Essential Skills for Mathematics', available on WJEC website.

## 6.2 Opportunities for use of technology

It is expected that candidates will have access to calculators and other appropriate technological aids during the course.

In the examination the following rules will apply.

**Calculators must be:**

- of a size suitable for use on the desk,
- either battery or solar powered.

**Calculators must not:**

- be designed or adapted to offer any of these facilities:
  - language translators,
  - symbolic algebra manipulation,
  - symbolic differentiation or integration,
  - communication with other machines or the internet.
- be borrowed from another candidate during an examination for any reason.
- have retrievable information stored in them - this includes:-
  - databanks,
  - dictionaries,
  - mathematical formulae,
  - text.

**The candidate is responsible for the following:**

- the calculator’s power supply,
- the calculator’s working condition.

## 6.3 Spiritual, Moral, Ethical, Social and Cultural Issues

This specification will enable centres to provide courses in Additional Mathematics that will allow candidates to discriminate between truth and falsehood. The mathematical models of the real world will naturally raise for discussion moral, social and cultural issues. Candidates will be required to reason logically and to consider the consequences of decisions.

## 6.4 Citizenship

This specification is designed to make a contribution to the development of the knowledge, skills and understanding of citizenship. Opportunities for addressing citizenship will arise naturally, particularly when candidates address problems in number.

## 6.5 Environmental Issues

The study of number and mensuration will give candidates the opportunity to discuss the various environmental issues facing society.

## 6.6 Health and Safety Consideration

Aspects of the work which require the use of ICT will allow candidates the opportunity to consider a variety of health and safety issues.

## 6.7 The European Dimension

Relevant examples, chosen by the teacher/student to illustrate mathematical concepts, will have a global, European and/or national context.